Development of Methods for Research of Flexibility of Electric Power Systems

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Abstract. This paper presents deterministic methods developed to study the flexibility of an electric power system. The methods based on the proposition that an electric power system is flexible if a power balance is maintained at the considered period of time. The developed methods are aimed at determining the combination of the largest loads which, when exceeded a little, disturb the power balance at the studied nodes. The paper presents two methods, which are: the brute-force optimization and the nonlinear optimization. The results of the first method are accepted as a reference for the verification of the nonlinear optimization output.

1 Introduction

In terms of control of an electricity system (ES), the flexibility of the ES that has generating equipment with specific maneuverability characteristics is closely related to its ability to maintain frequency and voltage in the system under conditions of uncertainty and variability [1]. Thermal and hydroelectric power plants, which can quickly ramp up and ramp down the load, provide flexibility of the ES on the generation side. A variety of load management techniques that have emerged owing to the development of new technologies solve the flexibility problem on the demand side. With the adoption of wind and solar farms, energy storage is becoming an important means of ensuring flexibility.

A prerequisite for ensuring the electric power system (EPS) flexibility is the available operating reserves in the system. The considered reserves or sources of flexibility are:

1. Operating reserves [2], [3], [4].
2. Demand management [5].
3. Energy storage systems [6].

Reserve is used in case of unplanned increase or decrease in load. The presence in the EPS of a large number of sources of variable generation (wind, solar) requires the placement of upward and downward reserves [2]. Authors [3] provide an overview of the operating reserves used in the USA and Europe. In [4] methodology for determining the minimum required volumes of reserves of Russia EPS is given.

Demand side management is applied to adjust residential load [7], service sector load [8] and industrial load [9].

Energy storage devices are used to store and deliver power during a certain period of time. Energy storage technologies are based on different physical principles. The following classification of energy storage devices is given on the site [6]:

• Mechanical: flywheels, hydraulic accumulators, pneumatic accumulators.
• Electric: capacitors and supercapacitors.
• Electrochemical: storage batteries, hydrogen fuel cells, nano-ion cells.

Author [10] describes Superconducting Magnetic Energy Storage (SMES), which store energy in a magnetic field created by a direct current in a coil with zero electrical resistance, cooled below a characteristic critical temperature.

Researchers in many countries are studying the issues of the flexibility margin, presence, and absence in the power system. There are currently probabilistic and deterministic methods for determining flexibility.

In [11] a deterministic method was proposed for determining the largest variation range of uncertainties at which the power system remains flexible for a specified time within acceptable cost. The flexibility metric is calculated by comparing the obtained range with the target range. In [12] the flexibility residual, which is the difference between the available flexibility and the expected load ramps for each observation and horizon is calculated. Then, the probability that the residual flexibility will be less than zero is determined, which means the probability of insufficient resources in the system. In [13] the flexibility of thermostatically controlled loads (TCLs) when integrating them into system level operation and control is calculated. Authors propose a geometric approach to model the aggregate flexibility of TCLs. The set of valid power profiles of individual TCLs is represented by a polyhedron. Aggregated flexibility is calculated as the Minkowski sum. The authors developed the optimization algorithm for approximating polyhedral by homotheties of given convex set represented by a virtual battery model.

The insufficient ramping resource expectation (IRRE) metric to estimate flexibility is calculated in [14]. For each direction and time horizon, a probability distribution of IRRE is formed.

This paper presents deterministic methods developed to study the flexibility of EPS. The structure of the article is as follows. The second section describes the modeling of EPS facilities flexibility and the modeling of the loads archive. In the third section, the ideas of methods, the objective function of calculating the maximum loads and constraints are prescribed. The fourth section provides a detailed description of the methods for calculating flexibility. The fifth section presents the research results. The sixth section is the conclusion.

2 Modeling the flexibility of EPS elements and load archive

Model of generator flexibility of a conventional station

The flexibility available from each generator is determined by the power that can be generated over the considered time horizon and is calculated by the formula [14]

$$F_g = V_{i+}*(t-(1-b)*S_i), \quad (1)$$

where V_{i+} is load ramp time (MW/min), t is the considered time horizon, S_i is the startup time (hour), b is the binary on-line variable, when a generator is on $b=1$.

Model of battery flexibility

The flexibility available from the battery is determined by the state of charge of the battery. If the battery is charged within the specified limits

$$SOC_{min} < SOC(i) < SOC_{max}, \quad (2)$$

then the power output is calculated by the formula:

$$F_B = P_{max}, \quad (3)$$

otherwise:

$$F_B = 0. \quad (4)$$

Model of system flexibility

System flexibility is defined as the sum of the flexibility available from all units of flexibility

$$F_S = \sum_1^m F_g + \sum_1^n F_B\ , \quad (5)$$

where m is the number of generators at conventional stations, n is the number of batteries.

Modeling of loads archive

The load at each given node *i* is calculated by the formula [11]:

$$P_i(z_i) = z_i P_i^{min} + (1-z_i)P_i^{max} \quad (6)$$

where $0 \le z_i \le 1$, P_i^{max}– the upper limit of load at node *i*, P_i^{min} – the lower limit of load at node *i*.

The archive of loads is formed according to the following algorithm:

1. The minimum and maximum values of the active load are set. The vectors P^{min} and P^{max} are formed $P^{\min} = (P_1^{\min}, P_2^{\min}, ...P_i^{\min}, ...P_R^{\min})$, $P^{\max} = (P_1^{\max}, P_2^{\max}, ...P_i^{\max}, ...P_R^{\max})$, where R – the number of given load nodes.
2. The vector z $z = (z_1, z_2, ...z_i, ...z_R)$ is set. The number of steps N, which determines the size of the archive is specified. The step of changing the load is calculated by the formula

$$step = 1/N\ . \quad (7)$$

Initial condition: z= (0) – is the zero vector, k =1 is the step number.

3. The value of load is calculated by (6).
4. $k = N*R$?, if yes, go to item 7, otherwise $k = k+1$, go to item 5
5. z^{k+1} is calculated

$$z^{k+1} = z^k + step. \quad (8)$$

6. Go to item 3.
7. Determination of all possible load values.
8. The end. Result: the archive of loads P^{LOAD}, the dimension of the archive is $[L \times R]$ where $L = C_N^R$.

3 Idea of methods. Objective function and constraints

This paper presents deterministic methods based on the proposition that an EPS is flexible if a power balance is maintained at the considered period of time. An increase in the load leads to a decrease in the flexibility of the system, this is why one of the key points in the analysis of the EPS flexibility is the availability of information about the maximum possible loads. The developed methods are aimed at determining the combination of maximum loads which, when exceeded a little, disturb the power balance at the studied nodes.

The objective function is the maximum of the sum of the differences between the predicted and simulated loads at nodes with uncertainty over a given period of time. It is written as follows:

$$\sum_{i=1}^{r}(\bar{P}_i - P_i(z_i)) = \sum_{i=1}^{r} \Delta P_i(z_i) \quad \to max, \quad (9)$$

where r – - the number of nodes with uncertainty.

For clarity of presentation of the constraints used to solve this problem, all nodes are divided into three types:

- Uncontrolled nodes. Generator nodes where control actions are not performed or load nodes at which there is no uncertainty P^{CONST};
- Controlled nodes. Generator nodes where the control actions P^{CA} are performed;
- Nodes with uncertainty. Load nodes at which power changes.

The constraints are as follows:

$$\Delta P_j = 0 \ , \tag{10}$$

$$P_{i-j} < P_{i-j}^{max} \ , \tag{11}$$

$$P_i^{min} < P_i^{CA} < P_i^{max}, \tag{12}$$

$$0 \le z_i \le 1 \, . \tag{13}$$

Where in (9) $\overline{P_i}$ – the forecast (pseudo measurement) of active power at node *i*, which has uncertainty; $P_i\ (z_i)$ is relationship between active power and value *z*, which is responsible for a change in the value of power at node *i*. Constraint (10) is the power balance at node j (any type of node), or the power balance at EPS, (11) is the constraint on line transfer capability; P_{i-j}^{max} is the capability limit of transmission line *i-j*, (12) limits the range of control actions at the controlled node, (13) is the constraint on the parameters of optimization.

4 Detailed description of the developed methods

The paper presents two methods for determining flexibility:
1. The method of brute-force optimization.
2. The nonlinear optimization.

A. The brute force optimization

The brute force optimization is used to process all combinations of possible loads in EPS to determine load combinations that, when slightly exceeded, make the system inflexible.
The brute force optimization algorithm is described below.
1. Start. The vector of injections is $P = (P^{CONST}, P^{CA}, P^{L})$. A load flow solution (LFS) is performed. $P^{ref} = P^{L}$, where P^{L} – load in the nodes with uncertainty at a given time. Initial conditions: $P^{rab} = P^{ref}$; $i = 1$.
2. The control action is performed P^{CA} in accordance with $P^{LOAD}(i)$.
3. The vector of injection is formed $P = (P^{CONST}, P^{CA}, P^{LOAD}(i))$.
4. A load flow solution is performed.
5. Has the process converged? If yes, then move on. Otherwise go to item 9.
6. Checking the constraints (formulas 10-13).
7. Have the constraints been satisfied? If yes, then move on. Otherwise go to item 9.
8. The vectors $(P^{rab} - P^{ref}) < (P^{LOAD}(i) - P^{ref})$ are compared. When the condition is met the vector $P^{LOAD}(i)$ is saved, $P^{rab} = P^{LOAD}(i)$. Euclidean distance and distance of Chebyshev are used to compare two vectors.
9. $i = i + 1$. $i = L$? If no, then go to item 2. Otherwise go to item 10.
10. The end. The result: $P^{LOAD}(i)$ – a combination of the largest loads in EPS which are possible under given condition.

B. Nonlinear optimization

Nonlinear minimization refers to the problem of nonlinear programming and is performed in Matlab. As result the values of optimization parameters that are used for calculation of active power P_i^{calc} at the nodes with uncertainty are determined. In this study the optimization parameter is z (formula (6)). Therefore, objective function (9) and constraints (10), (11) should be written using the parameter *z*. Constraint (12) is taken into account by the objective function.

The objective function

Each element of (9), taking into account (6), can be written as:

$$\Delta P_i(z_i) = P_i - P_i\ (z_i) = \bar{P}_i - P_i^{max} + z_i\left(P_i^{max} - P_i^{min}\right) = D_i + F_i z_i \tag{14}$$

$$\overline{P_i} - P_i^{\max} = D_i \ , \tag{15}$$

$$P_i^{\max} - P_i^{\min} = F_i \ . \tag{16}$$

D_i, F_i remain the constant values during the optimization process.

The objective function can be written in the following form

$$D_i + z_i F_i + \ldots D_R + z_R F_R \rightarrow max \tag{17}$$

and after excluding all constant values it has compact form:

$$\left(\sum_{i=1}^{R} -F_i z_i\right) - F_A z_A \rightarrow max. \tag{18}$$

Constraints

Power balance in EPS

$$\sum_{i=1}^{un} P_i + \sum_{j=1}^{n-un} P_j\left(z_j\right) = 0 \tag{19}$$

after some transformation can be written as follows

$$\sum_{1}^{n-un} F_j \ \ Z_j = \sum_{1}^{u-un} P_j^{max} + \sum_{1}^{un} P_i \ , \tag{20}$$

where n is the number of nodes in EPS, un is the number of uncontrolled nodes.

To form the balance and transmission constraints which need to ensure that all state variables are within their limits it is necessary to have the power flow values in the lines. In this study the power flows in the lines are calculated using PTDF (power transfer distribution factor) method [15]. PTDFs describe how active power flows in lines are changed if power injection in the node is increased or decreased.

The power transfer distribution factor in the line, which is limited by nodes *i*, *j*, is calculated in advance as follows

$$k_{i-j} = \Delta P_{i-j}/\Delta P_A(z_A), \quad (21)$$

$$\Delta P_A(z_A) = \sum_{i=1}^{R} \Delta P_i\,(z_i)\,, \quad (22)$$

where ΔP_A – an increase (decrease) of the active power at the node, where the control action is performed ΔP_{i-j} – an increase (decrease) of the active power flow in line *i-j*, ΔP_i – an increase (decrease) of active power at node with uncertainty.

For the problem of nonlinear optimization, the coefficients k_{i-j} are the initial data.

The power balance in nodes with uncertainty is compiled as a balance of power increments

$$\Delta P_i + \sum_{j=1}^{G} \Delta P_{i-j} = b, \quad (23)$$

$$\Delta P_{i-j} = \Delta P_A(z_A) k_{i-j}. \quad (24)$$

where ΔP_A – an increase (decrease) of active power at node *A*, ΔP_{i-j} – power flow increments, *G* –the number of adjacent nodes, b – convergence tolerance. Considering (14), (24) constraint in node *i* is written as:

$$F_A Z_A \sum_{j=1}^{g} k_{i-j} + F_i Z_i + (P_i - P_i^{max}) \sum_{j=1}^{G} k_{i-j} = b. \quad (25)$$

Active power flows in lines are monitored according to (11)

$$P_{i-j} + k_{i-j}\Delta P_A(z_A) < P_{i-j}^{max} \quad (26)$$

when transferring constant values to the right-hand side (taking into account (14) for i = A), constraint (26) has the form

$$F_A k_{i-j} z_A, < P_{i-j}^{max} - P_{i-j} - k_{i-j}(P_A - P_A^{max}). \quad (27)$$

C. Flexibility of EPS definition

Flexibility of EPS is defined as follows:

$$F_S = \sum_{i=1}^{R} (P_i^{calc} \quad - P_i^{forec}). \quad (28)$$

where P_i^{calc} – calculated value of active load at the node *i*; P_i^{forec} – forecast of active load at the node *i*.
In case $F_s > 0$, then EPS is flexible.

5 Case study

A. Describing a test schema and scenario

Calculations are performed on a scheme consisting of 5 nodes and 5 lines (figure 1). Nodes 3 and 4 are the nodes with uncertainty. Node 1 stands for a wind farm. Node 5 is a battery. Node 2 (conventional plant) is a controlled node where control actions are power generation required to ensure balance in the EPS, given the forecast of generation at the wind farm and the power supplied by the battery. Nodes 1 and 5 are considered uncontrolled.

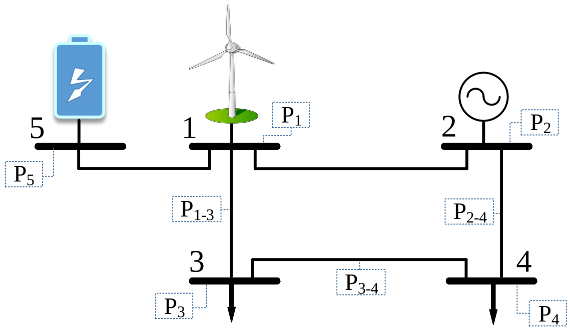

Fig.1. Test schema.

The calculations are performed according to the scenario: it is necessary to determine a combination of the largest loads at nodes 3, 4 four minutes ahead with known:

- forecasts of load values at nodes 3 and 4 (P^{forec}), which are assumed to be the lower limits of loads;
- forecast of active power output at the wind farm;
- forecast of active power output at the battery;
- maximum load values, which are the upper limits of loads;
- maximum value of active power generation at node 2;
- capacity limits of transmission lines.

It is assumed that it takes 4 minutes for the entire available reserve at the conventional plant to be switched on, and that the battery produces maximum power.

Table 1. Initial data (MW).

Number of nodes	P^{max}	P^{min}	P^{forec}
1			
2	32	20	20
3	37	23	23
4	23	13	13
5			

The flexibility of a 5-node EPS is calculated by two methods: the brute force optimization and non-linear minimization. The results of the first method are accepted as a reference.

B. The brute force optimization applying

Using this method, the vector of active loads is determined among 900 pre-created vectors that differs as much as possible from the forecasted loads when the following constraints are met: iteration convergence tolerance is 0.05 MW (0.05 MVAr), the upper limit of power generation at node 2 is 32 MW, active power flows in all lines should be within transfer capability. As a measure of the difference between the two vectors, two measures are used, which are: the Euclidean distance and the Chebyshev distance. As a result of applying this method, the load flow solution with the maximum possible loads at nodes 3 and 4 is calculated. Figure 2 shows the values of active power injection which are the results of the calculation of three load flow solution: the obtained loads at nodes 3, 4 are equal to the forecast loads (Steady state); the obtained loads at nodes 3, 4 are maximum loads in accordance with Euclidean distance (SSeuclidean) and Chebyshev distance (SSchebyshev).

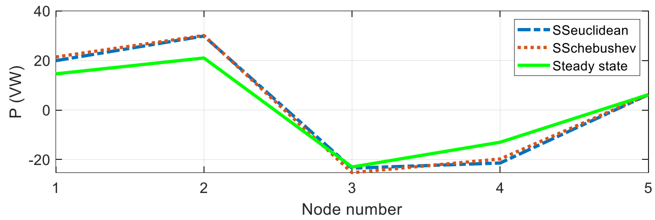

Fig. 2. The active power injections.

C. Nonlinear optimization applying

Solving the problem which applies the nonlinear optimization, developed in matlab, can be divided into several steps.

First step. Calculation of the coefficients (k_{i-j}) according to (21).

Second step. Description of the objective function and constraints in the equivalent forms which are suitable for the programs, developed in matlab.

The objective function

$$\left(\bar{P}_2 - P_2(z_2)\right) + \left(\bar{P}_3 - P_3(z_3)\right) + \left(\bar{P}_4 - P_4(z_4)\right) \rightarrow max \quad (29)$$

in equivalent form is written as follows:

$$-z_2F_2 - z_3F_3 - z_4F_4 \rightarrow max. \quad (30)$$

Similar transformations are performed for constraints.

EPS active power balance:

$$P_1 + P_2(z_2) + P_5 - P_3(z_3) - P_4(z_4) = 0 \quad (31)$$

has form:

$$F_2Z_2 + F_3Z_3 + F_4Z_4 = P_3^{max} + P_4^{max} - P_2^{max} - P_1 - P_5 . \quad (32)$$

Active power balance at nodes 3,4

$$\Delta P_3 + \Delta P_{3-1} + \Delta P_{3-4} = 0, \quad (33)$$

$$\Delta P_4 + \Delta P_{4-2} + \Delta P_{4-3} = 0, \quad (34)$$

are transformed into the equations:

$$F_2Z_2(k_{3-1}+k_{3-4}) + F_3Z_3 = (P_3 - P_3^{max}) + (P_2 - P_2^{max})(k_{3-1}+k_{3-4}) + b . \quad (35)$$

$$F_2Z_2(k_{4-2}+k_{3-4}) + F_4Z_4 = (P_4 - P_4^{max}) + (P_2 - P_2^{max})(k_{4-2}+k_{3-4}) + b . \quad (36)$$

Transmission constraints

$$P_{i-j} + k_{i-j}\Delta P_2(z_2) < P_{i-j}^{max} \quad (37)$$

are transformed into the following inequalities:

$$F_2k_{1-2}z_2, < P_{1-2}^{max} - P_{1-2} - k_{1-2}(P_2 - P_2^{max}) \quad (38)$$

$$F_2k_{1-3}z_2, < P_{1-3}^{max} - P_{1-3} - k_{1-3}(P_2 - P_2^{max}) \quad (39)$$

$$eF_2k_{1-5}z_2, < P_{1-5}^{max} - P_{1-5} - k_{1-5}(P_2 - P_2^{max}) \quad (40)$$

$$F_2k_{2-4}z_2, < P_{2-4}^{max} - P_{2-4} - k_{2-4}(P_2 - P_2^{max}) \quad (41)$$

$$F_2k_{3-4}z_2, < P_{3-4}^{max} - P_{3-4} - k_{3-4}(P_2 - P_2^{max}) \quad (42)$$

Optimization parameters constraints are:

$$0 \leq z_2 \leq 1, \quad (43)$$

$$0 \leq z_3 \leq 1, \quad (44)$$

$$0 \leq z_4 \leq 1. \quad (45)$$

The objective function in the matlab codes is: [z,fval]=fmincon(@funn,z0,ineq_l,ineq_r,A,B,zmin,zmax);

Function f= funn; $f = -F_2z_2 - F_3z_3 - F_4z_4$; initial approximation of optimization parameters are :$z0 = [0\ 0\ 0]$. For representing constraints, a compact matrix formulation is used.

Equality constraints (A, B) are:

$$\begin{array}{cccc} & \text{A} & & \text{B} \\ F_2k_{1-5} & 0 & 0 & (P_2 - P_2^{max})k_{1-5} \\ F_2(k_{1-3}+k_{3-4}) & F_3 & 0 & \bar{P}_3 - P_3^{max} + (P_2 - P_2^{max})(\sum_{j=1}^{G} k_{i-j}) \\ F_2(k_{4-2}+k_{3-4}) & 0 & F_4 & \bar{P}_4 - P_4^{max} + (P_2 - P_2^{max})(\sum_{j=1}^{G} k_{i-j}) \\ F_2 & F_3 & F_4 & \sum_1^{u-un} P_j^{max} + \sum_1^{un} P_i \end{array}$$

Inequality constraints (L, R,) are:

$$\begin{array}{cccc} \text{L} & & & \text{R} \\ F_2k_{1-2} & 0 & 0 & P_{1-2}^{max} - P_{1-2} - k_{1-2}(P_2 - P_2^{max} \\ F_2k_{1-3} & 0 & 0 & P_{1-3}^{max} - P_{1-3} - k_{1-3}(P_2 - P_2^{max}) \\ F_2k_{1-5} & 0 & 0 & P_{1-5}^{max} - P_{1-5} - k_{1-5}(P_2 - P_2^{max}) \\ F_2k_{2-4} & 0 & 0 & P_{2-4}^{max} - P_{2-4} - k_{2-4}(P_2 - P_2^{max}) \\ F_2k_{3-4} & 0 & 0 & P_{3-4}^{max} - P_{3-4} - k_{3-4}(P_2 - P_2^{max}) \end{array}$$

Third step. Optimization is performed. The result is a vector of optimization parameters (z) with given constraints.

Fourth step. Interpretation of the results. Calculation of the load values at nodes 3, 4 and the generation at node 2 according to the formula (6). Figure 3 shows the values of active power at nodes 2, 3, 4 before (SS) and after (SS max) optimization.

Fifth step. Analysis of the obtained state variables for being them within given limits is performed. If the result is negative, invalid state variables are assumed to be corrected.

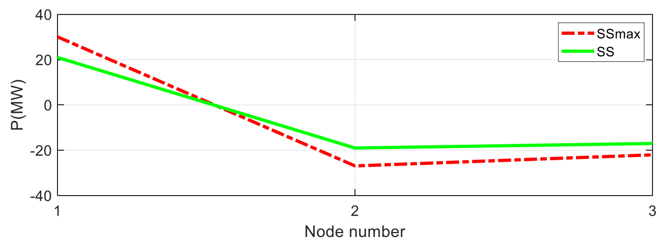

Fig. 3. Active power at nodes 2, 3, 4 before (SS) and after (SS max) optimization.

D. Analysis of results

The results obtained by two methods are summarized in table 2. The last line shows the flexibility calculated by (28) using different methods. Table 3 presents the values calculated by the formula:

$$\Delta F_{dist} = |F_{nonl} - F_{dist}| \quad , \qquad (46)$$

where F_{dist} – EPS flexibility (F_s table 2) calculated by the brute force method using Euclidean distance (8.8MW) or Chebyshev distance (9.3MW), F_{nonl} is EPS flexibility (12.9 MW, table 2) calculated by nonlinear optimization.

The active powers which are the result of three load flow solution problems and the result of the nonlinear optimization are shown in Figure 4 in a visual form.

Table 2. Result of calculations (MW).

	Initial data		Calculated data		
Number of nodes	**Forecast P_i^{forec}**	**Max P_i^{max}**	**Brute force P_i^{calc}**		**Nonlinear P_i^{calc}**
			euclid	**cheb**	
1	13.9				
2	21	30	31		30
3	-23	*37*	**-23.4**	**-25.5**	**-26.9**
4	-13	*25*	**-21.4**	**-19.8**	**-22**
5	6.2	6.2	6.2		
F_s			8.8	9.3	12.9

Table 3. Absolute difference between two values of flexibility.

	$\Delta F_{dist(i)}$ (MW)		$\sum \Delta F_{dist}$ (MW)
	3	4	
$F_{nonl} - F_{eucl}$	3.5	0.6	4.1
$F_{nonl} - F_{cheb}$	1.4	2.2	3.6

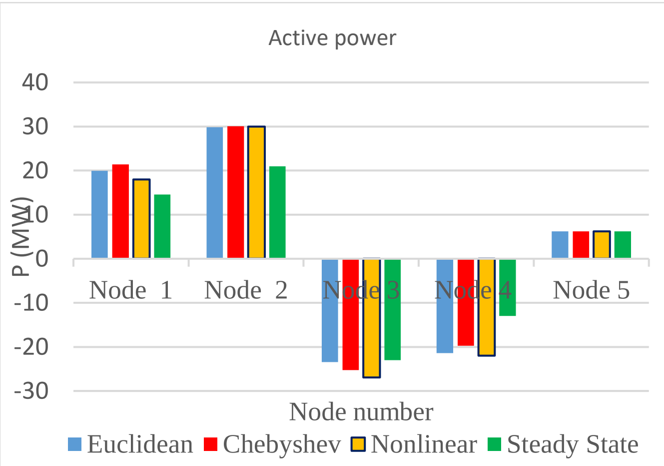

Fig. 4. Diagram of active power.

Analysis of the results of table 2 and Fig. 4 shows that

• Comparison of the given maximum loads (italics) with the maximum possible loads calculated by different methods (bold) shows that the loads calculated using nonlinear optimization are closer to the given maximum loads;

• The flexibility of the considered EPS is 12.9 MW (formula 28) according to non-linear optimization, 8.8 MW and 9.3 MW according to the brute force method when using the Euclidean distance and Chebyshev distance, respectively, as a metric.

Table 3 shows that the results of nonlinear optimization are closer to the results obtained by the brute force method in case of using the Chebyshev distance as a metric (4.1> 3.6).

It was revealed that when calculating load flow solution, where the result of nonlinear optimization is used as the initial data, it is necessary to add reactive power at node 3 for all variables are within given limits.

6. Conclusion

The article describes the methods for determining the flexibility of the electric power system: the method of the brute force and nonlinear optimization. In the brute force method, the Euclidean distance and the Chebyshev distance are used as a metric for comparing the two vectors. For nonlinear optimization, a function developed in matlab is used.

The analysis of the nonlinear optimization results is carried out. It is shown that the loads calculated using nonlinear optimization are closer to the given maximum loads. It is revealed that to ensure the balance of the reactive power of the EPS at the obtained load values, it is necessary to increase the reactive power at node 3.

A comparing analysis of the results showed that as the reference for the verification of the nonlinear optimization output should be used those results which were calculated by the brute force algorithm based on the metric of Chebyshev distance.

An algorithm has been developed to create an archive of loads required for the brute force method.

Acknowledgment

This study is supported by grant № 19-49-04108. "Development of Innovative Technologies and Tools for Flexibility Assessment and Enhancement of Future Power Systems”.

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